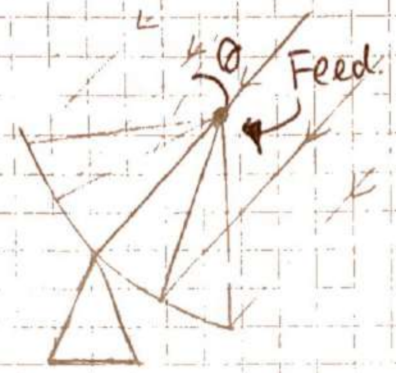
Crab Pulsar at JBO

30/01/2024



Schematic Diagram of a radio telescope.

* The telescope we use for this experiment is the 13m, "42-ft" radio telescope at Jodrell Bank observatory, Cheshire.

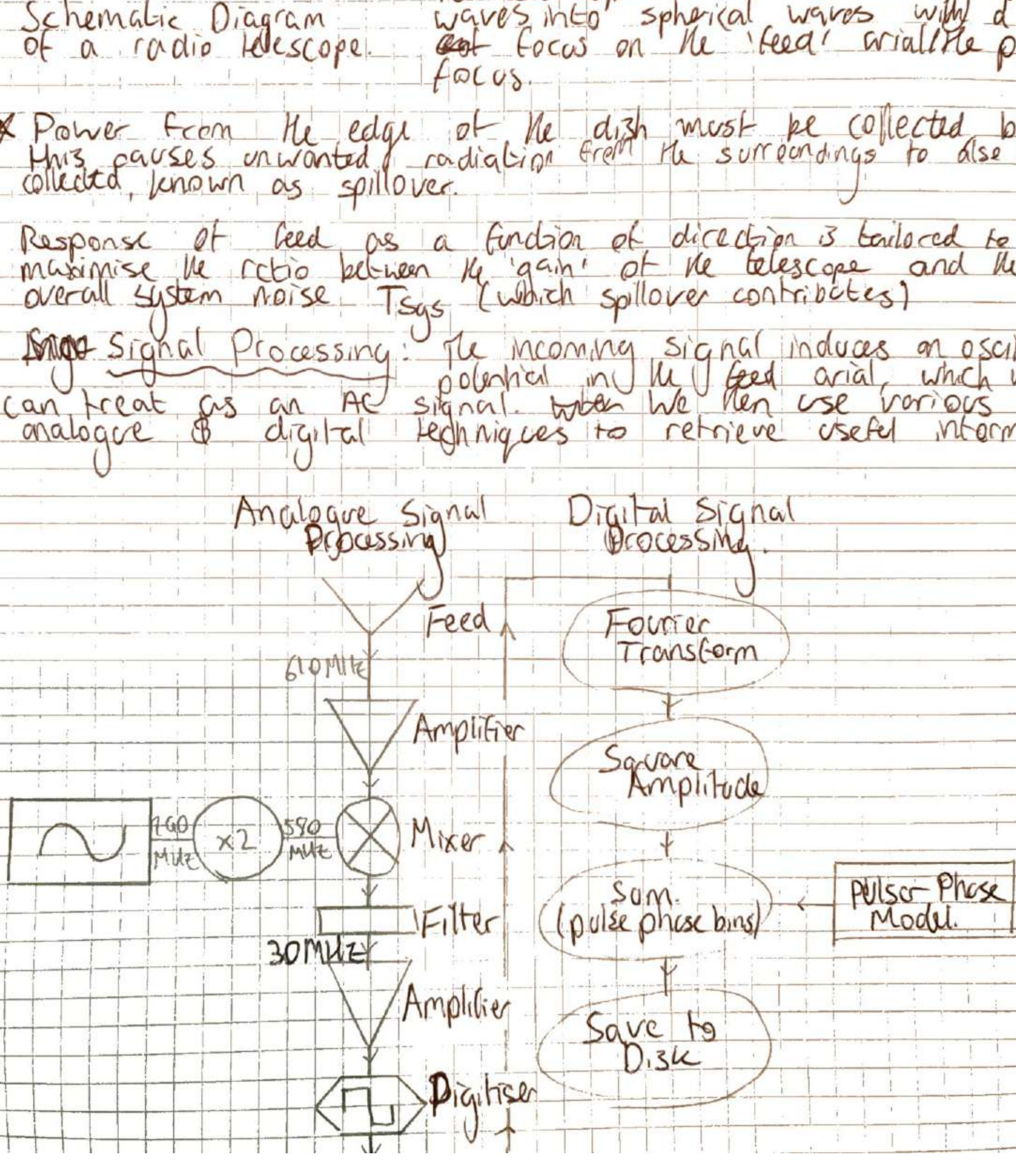
Optics: A parabolic dish focuses incoming radio radiation onto the "feed" arial at a focus point.

The telescope turns the incoming plane waves into spherical waves with a ~~flat~~ focus on the 'feed' arial/the prime focus.

* Power from the edge of the dish must be collected but this causes unwanted radiation from the surroundings to also be collected, known as spillover.

Response of feed as a function of direction is tailored to maximise the ratio between the 'gain' of the telescope and the overall system noise T_{sys} (which spillover contributes)

~~Sign~~ Signal Processing: The incoming signal induces an oscillating potential in the feed arial, which we can treat as an AC signal. ~~When~~ We then use various analogue & digital techniques to retrieve useful information.



~~*~~ The signal is mixed with a produced/synthetic oscillatory signal resulting in sum and difference frequencies.

$f_{diff} = f_{RF} - f_{LO}$, $f_{sum} = f_{RF} + f_{LO}$

f_{RF}: frequency of the generated/incoming e.m. radiation from the feed origin

f_{LO}: frequency of the Local Oscillator.

We then filter to keep only fdiff. These are more convenient to pass along ~~the~~ long cables from the telescope to the observing room.

The signal is then digitised. We are interested in signal as a function of frequency.

→ Take a small segment of the voltage data and compute a Fourier transform to generate 40 frequency channels.

$\frac{10MHz}{40} = 250kHz$ bands / channel width ?

Squaring the amplitude of the complex numbers from the signal gives us the power for each frequency channel.

→ Gives power as a function of frequency every $2\mu s$ → $\left(\frac{40}{2\times10MHz}\right)$

The two comes from the two polarisation states.

For observations of known pulsars we can compute a 'histogram' of power vs pulse phase (rotational phase of star)

Calibration & Noise Temp

- All signals will compose of random noise
- Characterised by 'noise temp' measured in kelvin
  → a mean noise power available from a resistor at T.

* Equipartition: $\frac{kT}{2}$ J of energy associated with each "degree of freedom" in the system.

2-polarisation states so power/unit bandwidth = $kT\,[WHz^{-1}]$

Over a bandwidth B, total power = $kTB\,[W]$

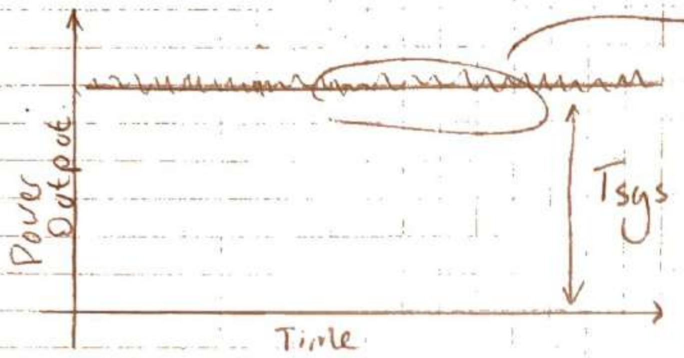
→ Calibrations made on "hot and cold" loads: resistors at well defined temperatures.

After amplification a detector (responds to the square of the input voltage) → power integrated over a certain period

→ Random fluctuations end up in the signal $\propto \frac{1}{\sqrt{B\tau}}$

τ: time over which signal is averaged.

∴ Accuracy improved by increasing signal bandwidth B & the averaging time, τ



→ Fluctuations $\propto \frac{T_{sys}}{\sqrt{B\tau}} = \frac{\sqrt{2}\,T_{sys}}{\sqrt{2B\tau}}$

$T_{sys} = T_{source} + T_{sky} + T_{spillover} + T_{rec}$

↓ (T_{source}) Contribution to the signal from the desired source (only on during the pulse so ignore)

T_{sky}: noise contribution from the ~~noise~~ sky around the source

$T_{spillover}$: noise contribution from the ground & area around the telescope

T_{rec}: noise contribution generated by receiver electronics.

In this exp: $\left.\begin{aligned} T_{sky} &= 20K \\ T_{spillover} &= 10K \\ T_{rec} &= 100K \end{aligned}\right\}$ T_{rec} highest source of random noise

Relationship between flux density of source & from the sky & equivalent noise temp characterised by gain/sensitivity

$[K\,Jy^{-1}]$ → $1\,Jy = 10^{-26}\,W m^{-2} Hz^{-1}$ $\quad [Jy^{-1}] = [10^{26} \frac{s}{J} m^2 \cancel{s}]$

→ Power per unit area per unit bandwidth

$G = \frac{\eta A}{2k_B}$ A: physical area, η: aperture efficiency = 55%

Gain for the "42-ft" telescope

→ 12.8m ∴ $A = \frac{\pi d^2}{4} = 128.7\,m^2$

$G = 2.56 \times 10^{24}\ m^2 K J^{-1}$ ∴ $0.0256\ K\,Jy^{-1}$

System equivalent flux density; S_{SEFD}: the flux density which would induce an increase in system temp by rms fluctuations in T

as $G S_{SEFD}$ = fluctuations

$$S_{SEFD} = \frac{\text{Fluctuations}}{G} = \frac{T_{sys}}{G\sqrt{B\tau}}$$

$$S_{SEFD} = \frac{T_{sys}}{G\sqrt{B\tau}} \times \frac{1}{\sqrt{n_p}}$$

↳ number of polarisation channels, n_p.

* Observation of B0329+54 14:26:09 } File stored on system.
→ Integration time; 600s, 10mins.

Minimum integration time required to detect 1Jy source in the 42-ft telescope.

~ 0.78 seconds

S_{SEFD} = Source flux density to be detected.

$$1\ Jy = \frac{T_{sys}}{G \cdot \sqrt{n_p B \tau}} \rightarrow T_{sys} \sim T_{sky} + T_{spillover} + T_{rec}$$

$$\therefore \tau = 0.78 \text{ seconds if } n_p = 2$$

Considered bright for extragalactic sources.

S_{SEFD} - Noise signal

S - Flux density of the source.

$$\frac{S}{S_{SEFD}} = snr = \frac{GS\sqrt{n_p B \tau}}{T_{sys}} \cdot \sqrt{\frac{P-W}{W}}$$

P: Pulsar period
W: Pulse width

* B is actually 5×10^6 Hz instead of 10×10^6 Hz because there exists a filter on the highest and lowest frequencies to eliminate them.

What do we expect snr for B0329+54?

S at 1400MHz = 203mJy $\quad S \propto f^{-1.4}$

→ Our f is at 611 MHz

$$\therefore \frac{S}{203} = \left(\frac{611}{1400}\right)^{-1.4} \Rightarrow S = 648\,mJy$$

→ Integration time 3s required. (For the peak to be above the fluctuations & recognisable).

$$\text{snr expected: } 13.98\sqrt{\frac{P-W}{W}}$$

$P = 0.714520$ s
$W = 6.6 \times 10^{-3}$ s

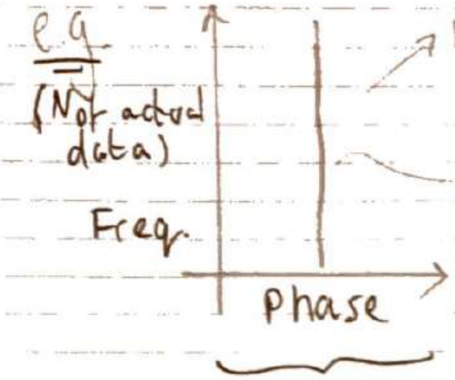
$\therefore$ SNR = 144.8 (Actual snr for 583s ≈ 33)

* Performed observation of B0531+21 at ~~14:39:07~~ 15:36:37
→ Integration time: ~~1410s~~ 1196s
→ Minimum integration time: ~645s.
→ SNR: ~ 4.27
→ Actual SNR = 9.32

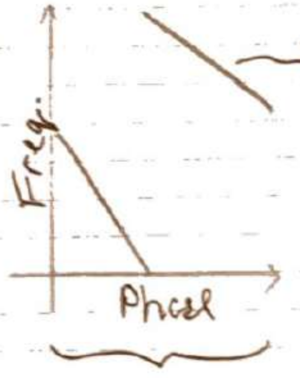
1400MHz : 14mJy
→ S at 611MHz : 44mJy

P: 0.033392s
W: 3×10^{-3} s

Both of these pulsars return some structure of graphical data



B0329+54



e.g... B0531+21
Crab Pulsar

→ Single pulse but different frequencies arrive at different times due to dispersion

Next week at Jodrell we will make more pulsar observations with high SNR

Dispersion

→ Plasma in the Interstellar medium is dispersive.
→ Frequency dependent, causes the tilt in the phase/freq graph
Must correct for this → de-disperse.

Dispersion Measure, $DM = \int_{pulsar}^{earth} n_e \, dl$. → smears sharp pulses.

Time to travel across the medium: $T = \int_0^L \frac{dl}{v_g} = \underbrace{\frac{L}{c}}_{\text{Non dispersive travel}} + \underbrace{\frac{e^2 \overbrace{\int_0^L n_e \, dl}^{DM}}{2\pi m c f^2}}_{\text{Dispersive addition}}$

The delay $t_1 = \frac{DM \rightarrow cm^{-3}\,pc}{2.410 \times 10^{-4} \, f^2_{MHz}}$ → basically constant.

Rough distance estimate to B0531+21. MHz Assume $n_e = 0.03 \, cm^{-3}$

DM = 55.5 gave a straight pulse across all frequencies.

$\therefore L n_e = DM$

where L = distance to pulsar.

$\therefore DM = L n_e = 55.5 \, cm^{-3} pc$ $n_e = 0.03 \pm$

$n_e = 0.03 cm^{-3} \rightarrow$ 1850pc.! Very close to accepted distance to crab pulsar.

$t_2 - t_1 = 4.15ms \, DM \left[(\nu_1/GHz)^{-2} - (\nu_2/GHz)^{-2} \right]$

$\therefore DM = (t_2 - t_1)/(4.15ms) \cdot \left[\left(\frac{\nu_1}{GHz}\right)^{-2} - \left(\frac{\nu_2}{GHz}\right)^{-2} \right]$

No errors yet. Must calculate programmatically.

Testing this for B0531+21
(0985.89)

$\nu_1 = 609.625 \, MHz$ $t_1 = 11.1 \times P \rightarrow 0.03342 s / 1024$

$\nu_2 = 612.125 \, MHz$ $t_2 = 164.4 \times P / 1024$

$\Delta t = 5.37 / 1024 = 5.24 \times 10^{-3}$

$\therefore DM = 57.62$

$$\Delta DM = \frac{(\sigma t_2^2 + \sigma t_1^2)^{1/2}}{4.15 \times 10^{-3}} \left[\left(\frac{\nu_1}{GHz}\right)^{-2} - \left(\frac{\nu_2}{GHz}\right)^{-2} \right]^{-1}$$ } 46

$\sigma t_1 =$ → error in arrival time. → error either side.

measurements taken every $2\mu s$ ∴ $\sigma t_1 = \pm 1 \mu s$

1×10^{-6}

$\Delta DM =$

∴ DM of B0351+21 "Crab Pulsar" = 57.62 ± ~~3.74~~

Pulse fits a gaussian with $\sigma = 0.07 \times P$ $= 2.37 \times 10^{-3} s$

Distance to this pulsar = 1921 ± ~~123~~ Pc

— Quoted Distance ...

(Rebecca Lin et. al. 2023) → d = 1900 ± 200pc.

Trying again for pulsar ~~B2020~~ B1933+16

DM =

$t_1 = 0.887 \times 0.3587$ } Period in s
$t_2 = 0.935 \times 0.3587$

$\nu_1 = 610 MHz$
$\nu_2 = 613 MHz$

$\Delta t = 0.0172 s$

$$DM = \frac{0.0172}{4.15 \times 10^{-3}} \left[(0.613^{-2}) - (0.61)^{-2} \right]^{-1}$$

$= 157.95 \pm ?$

d = 5265 pc.

Given Value

158.53 ± 0.05

→ within 2stdv

B1929+10

$t_1 = 0.385 \times 0.22652$ $\nu_1 = 0.612$
$t_2 = 0.337 \times 0.22652$ $\nu_2 = 0.610$

→ DM = initially : 21.79
then ... : 24.84

→ off by a pretty significant factor. which is interesting as the method is the same

Uncertainty:

→ Uncertainty comes in width of the pulse, as a wider pulse has more uncertainty in its arrival time.

psrweb.jb.man.ac.uk/lab/42ft.

Crab Pulsar Specifics ↳ Location of data retrieval site

We took a really long (10,250s integration time) data measure of the crab pulsar, and used our own written python script to calculate the dispersion measure DM.

→ This gave us a DM of 55.397 ± How to do errors?

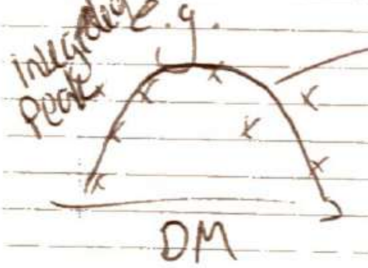
DM calculation:

→ to find the DM we calculate the maximum intensity in the integrated profile for a large range of DMs (i.e. 0–500 at 0.5 intervals)

→ The DM with the highest maximum gave us a rough estimate - ie- 55.5 for the Crab pulsar.

→ We then "zoomed in" to a region of ~ ±5 pc cm^{-3} then linearly fit a quadratic curve to the points; the variance on the coefficients can be used to figure out the error on the DM

e.g.



→ fitted quadratic $ax^2 + bx + c$

*our algorithm calculates the highest (peak) for a range of DMs at roughly 0.5 intervals, and fits a quadratic to the points to find the DM.

→ for 10,250s int time of Crab pulsar.

→ $a = -2.459 \pm 0.298 \times 10^{-4}$
$b = 2.782 \pm 0.332 \times 10^{-2}$
$c = -7.584 \pm 0.918 \times 10^{-1}$

max/min at $2ax + b = 0$

$$x = \frac{-b}{2a} = 56.57$$

$$\sigma DM = \sqrt{\left(\frac{\sigma b}{b}\right)^2 + \left(\frac{\sigma a}{a}\right)^2} \times DM = \pm 9.62$$

$DM = 56.6 \pm 9.6$

$$\sigma_x = \sqrt{\left(-\frac{1}{2a}\right)^2 \sigma_b^2 + \left(\frac{b}{2a^2}\right)^2 \sigma_a^2} = 9.62 \quad \underline{\text{as before}}$$

$$\simeq \sqrt{\left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2} \times DM$$

(ISM ne = 0.03) $\therefore$ DM crab = 56.6 ± 9.6. pc cm^{-3}

: This can give us an error on the distance.

$\rightarrow$ Distance to ~~gr~~ crab. = $\underline{1890 \pm 320 \text{ pc}}$

* Lets do to more significant figures by using the algorithm in Python

$\rightarrow$ DM = 56.5602 ± 9.6205 pccm^{-3}.
Dist = 1885.3414 ± 320.6842 pc

$\rightarrow$ Getting the data to be fitted over a more representative & symmetrical portion of the data can reduce this ~~to~~

DM = 56.6989 ± 5.3427 pccm^{-3}
Distance = 1889.9645 ± 178.0910 pc

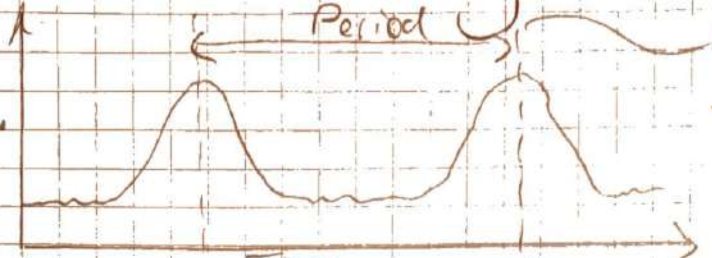
* The majority of this uncertainty comes from the method at which it is calculated. The data to which the curve is fitted to contains a lot of variation
* We have been told that Errors on DM are notoriously difficult to calculate & so we decided to leave this reasonable error of ~20%.

* We decided on the 4th day to spend the morning performing some data reading using the 42ft telescope of bright pulsars in the sky
$\rightarrow$ Later in this lab book we will go to find the DM and distance of these pulsars.

* Many of our measurements in the first ~~few~~ lab day had very low integration times of ~600s, meaning that small ~~a~~ & temporary bursts of interference ended up ~~with~~ causing large problems in the calculation of DM using the algorithm we ~~sapa~~ came up with.

* Will return to this segment once data has been taken.
* I also made improvements to the Python notebook which is stored ~~on our~~ our google drive
"Dispersion and data visualisation.ipynb"

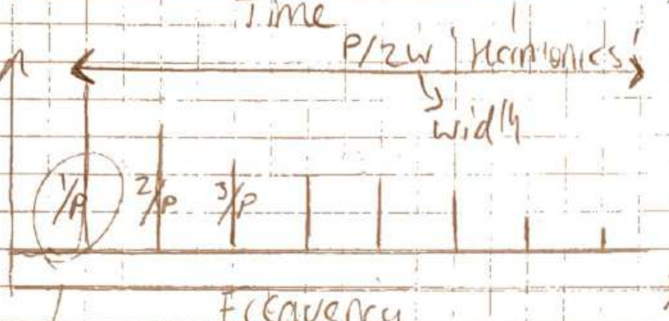
Fourier Transforming (Then Folding) to find Period & increase SNR.



▷ we want to find this.

→ We can use a fourier transform to transform this data into frequency space

* Once the period is known from the distance between harmonics in the transformed data, it can be "folded" such that the SNR (signal-noise ratio) is increased



First peak location can tell us the frequency.

→ This is due to the noise being random but the signal being predictable, so the signal height will increase faster than noise.

* had some initial issue about the frequency units on the x-axis in the transformed data

* found equation: $\text{frequency} = \dfrac{\text{bin number}}{dt \cdot N}$

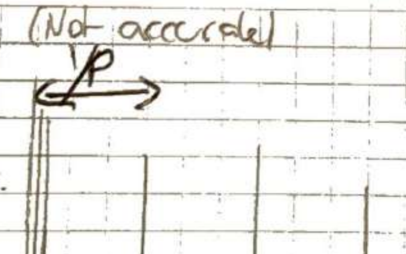
dt: time between each data point i.e sampling period

N: number of bins i.e. number of times data was taken in the data set.

* New python notebook: "Searching for Pulsars.ipynb"

Lets take "psr1.dat" for example → needed to be read in with as a string of 8bit integers. a seperate "psr1.hdr" gave information about the sample rate.

e.g (Not accurate)



These first few peaks are interesting but they cannot be the period
* Need to find out why they are there.

$\frac{1}{P} \sim 5.3\text{Hz}$

$\therefore P = 0.19\text{s}$ period, seems realistic but we should verify.

* only took positive frequencies, negative would just be symm about $x=0$.

* Power is from ~~mod~~ |absolute| value of the resultant complex numbers.

* This will be done algorithmically to calculate the uncertainty (width) on the pulse period.

Interestingly for PSR1, there is a huge spike at a frequency of ~50Hz. I don't believe that this is the pulsar pulse.

How many harmonics do we have for psr1?

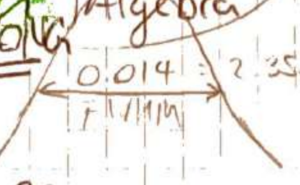
→ ~28 (some are super feint and some are extremely potent.)

$28 = \frac{P}{2W}$ rearrang for W $\quad \frac{28}{P} \times 2 = \frac{1}{W}$

$\# \text{Harmonics} = \frac{P}{2W} \therefore W = \frac{P}{2 \times NH}$ $\quad W = \frac{2P}{28} = 0.014\text{s}$ width

$\sigma = \frac{\cancel{2\sqrt{2\ln 2}}}{\cancel{\times NH}} P \cdot 2 =$ ~~[illegible]~~ redid Algebra

for a single pulse. ~~W~~



∴ for a gaussian width gives us an uncertainty on the arrival time of a pulse.

Not all pulsars have a gaussian pulse profile but many of them do.

(Australia Telescope National Facility)

FWHM ~~[illegible]~~ A.K.A Width $= 2\sqrt{2\ln 2} \cdot \sigma =$ ~~[illegible]~~ 2.355σ

∴ σ here $= 5.76 \times 10^{-3} = 5.8\text{ms}$

Error on arrival time

→ once at start and end of every pulse.

$\frac{2 \times N\text{-pulses} \times \sigma}{\text{~~[illegible]~~}}$ } can't be dep. on N-pulses as this should improve accuracy ∴ ÷ Npulses and they cancel out

$2 \times N\text{pulses} \times \sigma$ - error on ~~arrival~~ pulse dist./period. time of all pulses

$\frac{2 \times \cancel{N\text{pulses}} \times \sigma}{\cancel{N\text{pulses}}}$ - error on ~~arrival~~ pulse dist. time of a single pulse.

∴ $2\sigma =$ error on ~~arrival~~ period time -

$\frac{1}{\Delta P}$ $\quad \frac{1}{P} = \text{freq.}$

∴ P of psr 1 = 0.19 ± 0.01 seconds

→ more sig figs from algorithm for period.

$P = 0.18963 \pm 0.01433$ s $(\sim \pm 7.6\%)$

↳ from first peak in freq. space

Width $\boxed{\text{wrong}}$ ✗

$W = \frac{2P}{N\text{peaks}}$

$N\text{peaks} = 27$

$W = 0.01405$

$\sigma = 7.166 \times 10^{-3}$

$2\sigma = 0.01433$

Assuming that width can be used to compute the error in this way, which I am not entirely convinced is correct, ~~[illegible]~~

Ask about this on 05/03/24

pulsar	period (s)	# Harmonics
psr1	0.1896 ± ~~0.0143~~	27
psr2	0.4199 ±	24
psr3	0.7482 ± ~~0.0235~~	27
psr4	0.2990 ± ~~0.0127~~	26
psr5	0.7144 ± ~~0.0243~~	25
psr0	0.2265 ±	10

different / better
Will add errors later once I better understand them.

* PSR4 is interesting as it appears to have two different forms of harmonics?

→ Period found from measuring harmonic spacing & distance in frequency space to the zeroth harmonic.

* Ask: when do we say that there are no more harmonics? & what does it mean when frequency spacing doub

* Returning temporarily to DM measures

→ DM = 7.07 ± 1.09 $cm^3 pc^{-1}$
Distance = 238. ± 36 pc

{B1929+10}
Integration Time: 3600s
1hr

given DM on this pulsar from ATNF: 3.18

* Quite far off but our algorithm works much less efficiently for extremely low values of Dispersion Measure.

→ For shorter integration times, we couldn't find any value for DM on this pulsar and sometimes recovered a -ve DM.

DM = 33.4 ± 11.0 $pc\,cm^{-3}$
Distance = 1112.6 ± 367.3
1110 ± 370 pc

{B2020+28}
Integration Time: 3610s

DM on database ATNF: 24.6 $pc\,cm^{-3}$

---

* This one is poor Data.

{B2016+28}
Integration Time: 5420s

→ Data leads to the suggestion that DM ~ 88 but it is not even close to this.

→ Visually appears as if the DM should be 88 $cm^{-3}pc$ instead of the ATNF given 14.2 $cm^{-3}pc$

pulsar	~~period(s)~~	period (s)	width, W, (s)	# Harmonics
psr0	~~0.2265~~	0.2265	0.0113	10
psr1	~~0.4199~~	0.1896	0.0035	27
psr2	~~0.7482~~	0.4199	0.0087	24
psr3	~~0.2990~~	0.7482	0.0139	27
psr4	~~0.7144~~	0.2990	0.0025	26
psr5	~~0.2265~~	0.7144	0.0143	25

* Width was wrong on previous pages. $W = \frac{P}{2N}$ → # of Harmonics

$2\sigma = \frac{2W}{2\sqrt{2\ln 2}}$ } potential contender for ΔP ?

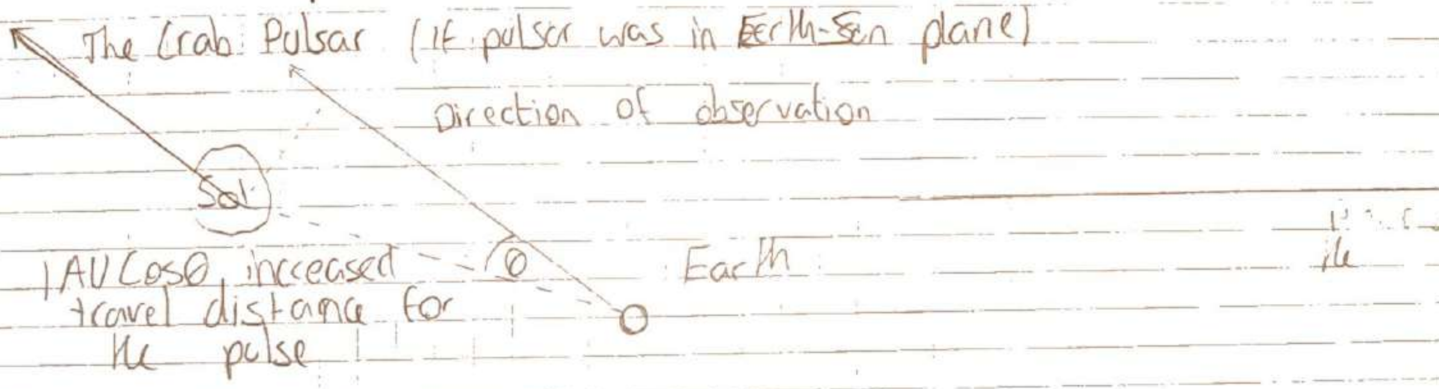
~~*~~ Still not convinced that what is labelled as ΔP here is actually how we calculate the error on the period.

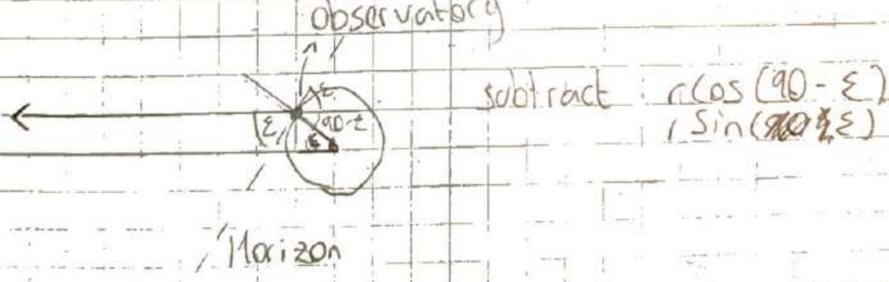
pulsar	ΔP (ms)
0	4.80
1	1.49
2	3.69
3	3.90
4	3.18
5	6.07

Pulse arrival times from the Crab Pulsar

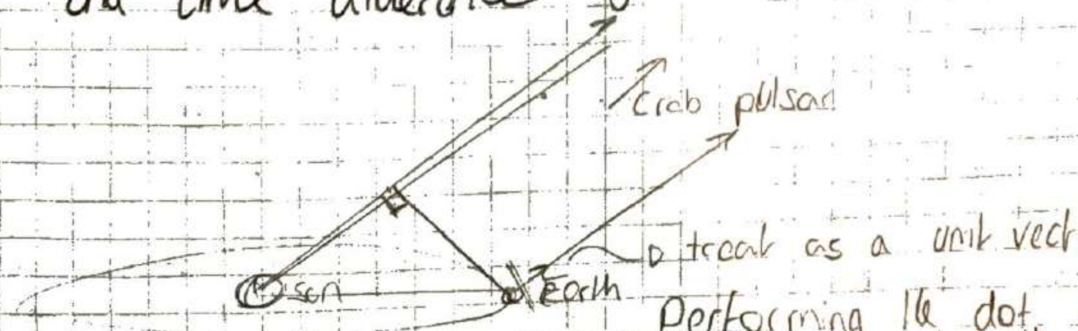
Raw arrival times are given in MJD, which ~~starts from~~ is * Final part. number of days since midnight Nov 17 1858

les. → Easy to retrieve raw pulse arrival times BUT they must be corrected for motion of observatory around the sun. i.e. pulse train should ~~be~~ appear to be collected from a fixed point in space.





* Now lets write a program to calculate the distance and time difference



Performing the dot product between the direction to the pulsar and the ~~directio~~ vector between Earth and the Sun will give the total difference

Using our dedispersion code and averaging over frequency granted use a list of ToA (time of arrivals) for each sub integration (vertically).

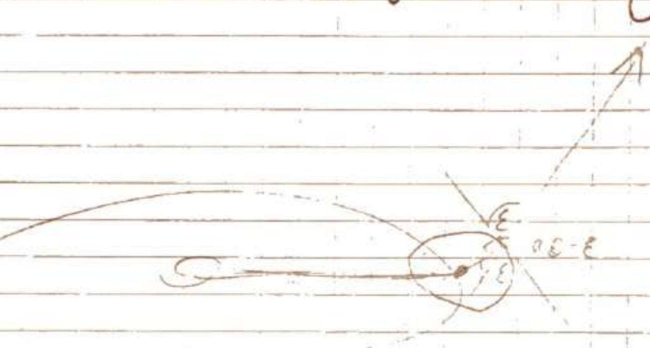
Conversion of equitorial (Ra, Dec) to (Alt, Az coords)

Lovell location : 02:18:25.74 (West)
($42-ft) 53:14:10.50 (North)

Lets define a plane with the Sun, earth $ pulsar

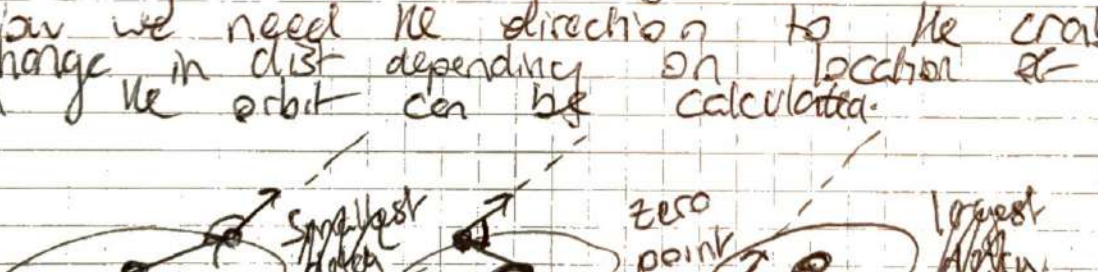
ε above the horizon is the same as $\varepsilon+90$ = Angle between distance to sun vector and pulsar direction vector.

How do we programmatically define a "pulsar-direction-vector"



So far, at the beginning of Day 7 we have a program which can interpolate the x, y, z of the barycentre for any given ToA.

Now we need the direction to the crab and so change in dist depending on location of the earth in the orbit can be calculated.



} ∴ We must ~~change~~ multiply by ~~-ve~~ one so that we subtract time

Asking for dot product so we need a vector which points in the direction of the crab pulsar.

→ such that we have a time when the pulse arrived at the barycentre

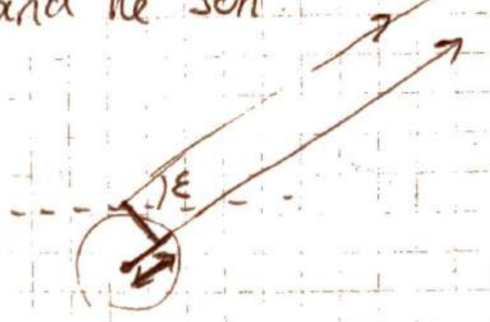
We have x, y, z defined

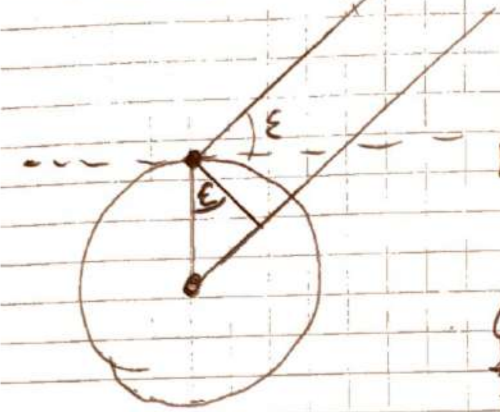
ICRS - International Celestial Reference Frame.

Origin is defined at the barycentre.

★ Converted the direction to the crab pulsar to an ICRS x, y, z unit vector, then performed the dot prod with the earth's coordinates
→ this gave an array of the distance difference between the earth & the pulsar based on the time of year / pos. of earth around the sun

Now lets do earth delay





$$\text{Earth delay} = \frac{R\sin\varepsilon}{c}$$

R → radius of the earth

→ Always reduces arrival times as we can only observe when we are pointing towards the pulsar.

always add, as ~~this~~ we want to find the time that

★ When we took our measurements the earth was closer to CP than the sun so Roemer delay is positive.

Lovell JBO

o → actual TOA

↓ Add Earth Delay.

o → time pulse arrived at centre of earth

↓ Add Roemer Delay (or subtract if sun is between crab & earth)

o → time pulse arrived at barycentre

↳ which is what we're interested in.

→ all toas have to appear

toa → time of arrival

t_0 → ref. epoch

P → Guess period

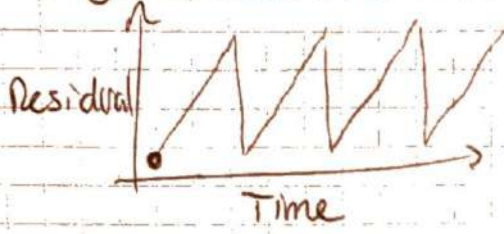
$$\text{Number of pulses} = \frac{toa - t_0}{P}$$

↳ If this is non integer the remainder is the residual.

i.e $\frac{t - t_0}{P_{guess}} \% P_{guess} = \text{residual}$

% → modulo operator in python.

Rough Image of residuals looks like:



→ linearly falls out with time until the residuals add up to one period, then it jumps down as the remainder would be 0

→ We want to find a period where these residuals are horizontal, ideally at zero.

This gradient is surely related to the difference in the period and the actual period.

residual is difference between regular pulse and actual

• • • • • • • regular pulse
→ time

e.g. Pulse
0
10
20
30

For each rotation our guess period P_G is off by ΔP

over time this coagulates at a rate of

$\frac{P - P_G}{P}$ per pulse per unit time

$1 - \frac{P_G}{P}$ how much it "falls out per unit time"

i.e. size of residual = Gradient

$\frac{P_{guess}}{(1 - \text{Gradient})} = P_{real}$ → Lets try this.

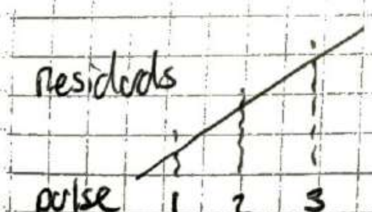
→ Didn't work lets have another think about it

if we are underestimating P

e.g.

P		P_{guess}		Residuals
10	−	9	=	1
20	−	18	=	2
30	−	27	=	3

} fall out ~~of~~ $ increase.

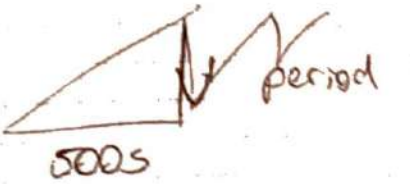


gradient = 0 ⇒ if $P_{guess} = P_{real}$

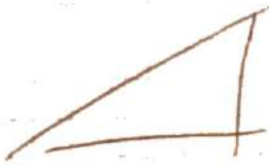
e.g in 10 seconds we expect

~~$\frac{10}{P}$ pulses but see~~ $\frac{10}{P_{guess}}$

gradient tells us how much the period guess falls out with time. ie how many pulses.



gradient $= \frac{P \cdot \cancel{\ }}{T}$



residual height (Period ~~x~~ − Pguess) ~~Avg~~ N

$$N = \frac{t - t_0}{P}$$

$$(P - P_a)\cdot\left(\frac{t - t_0}{P}\right) = y(t)$$

e.g t> y(9.89 $\quad \left(1 - \frac{P_a}{P}\right)\cancel{(t - t_0)} = \underbrace{\frac{\text{residual height}}{t - t_0}}_{\text{gradient}}$

$$1 - \frac{P_a}{P} = \text{Gradient}$$

$$1 - \text{Gradient} = \frac{P_a}{P} \qquad P = \frac{P_{guess}}{1 - \text{Gradient}}$$

First period accurate estimate: 0.033824965660821105

~~A~~ Find out what it means to fit a quadratic to the residuals.

Some Lab Report Research

- quadratic shape of residuals is when period guess is right but ~~g~~ doesn't account for $\dot{P}$ (spin up/spin down)
- TEMPO & TEMPO2 are often used for pulsar timing,
  → PINT is a python alternative but using modern python libraries rather than ancient dependencies.
- Crab pulsar is J0534+220. → can obtain a .par file online.

In the end, we used ~~the~~ 6 Crab Pulsar observations:

Crab Pulsar J0534+2200

		Integration Time	
1:	13/02/2024	10250s	~ 2weeks
2:	27/02/2024	10865s	~ 2weeks
3:	11/03/2024	32118s	~ 1day
4:	12/03/2024	9039s	~ 2weeks
5:	28/03/2024	32128s	~ 2weeks
6:	08/04/2024	32135s	

★ 1 day difference in obs. 3 & 4 was very useful in fitting for the period (F0) before adding in the rest of the data to fit for (F1).

→ Not enough data to effectively find F2 ($\ddot{P}$) (Lyne & Graham-Smith)

~~Using PINT~~ With these observations & PSRCHIVE we could use the PGS method (Taylor et. al. 1992) to find TOAs for each sub-integration (frequency averaged)

Once fitted using PINT & PINTk the following derived parameters were obtained

Period: $(0.03339375 4 \pm 0.000000080)$s

$\frac{d\,Period}{dt}$, $\dot{P}$ or Spin-down rate: $(4.1837 \pm 0.0016) \times 10^{-13}$

Characteristic Age: 1250 yrs (breaking index 3)

Surface Magnetic Field: 3.78×10^{12} G

Magnetic Field at light Cyl: 9.21×10^{5} G

Where are these derived parameters from: ?

★ Can improve errors by using FDM Monte Carlo method

→ PGS is a template matching Fourier Phase Gradient Scheme method where

$$\sigma \propto \frac{1}{\sqrt{N_{pulses}}}, \quad N_{pulses} \propto T_{integration}.$$

→ potentially add multiple 800second sub-integrations to improve S/N and contain more pulses in one TOA calculation.

Pulsar Age + Surface Magnetic Field

classical electrodynamics: magnetic ~~moment~~ dipole with moment $m_\perp$, rotating with ang. velocity $\Omega = 2\pi\nu$ radiates a wave at freq. Ω with total power: $\frac{2}{3}m_\perp^2\Omega^4 c^{-3}$

$$P_{rad} = \frac{2}{3}m_\perp^2\Omega^4 c^{-3} = \frac{2}{3}\underbrace{(BR^3\sin\alpha)^2}_{m_\perp}\underbrace{\left(\frac{2\pi}{P}\right)^4}_{\Omega}c^{-3}$$

$m_\perp = BR^3\sin\alpha$ (uniformly magnetised sphere radius R, surface mag field strength B)

This EM radiation will appear at a very low radio frequency

$\nu = \frac{1}{P} < 1kHz$ → cannot propagate through surrounding ionized nebula

This magnetic dipole radiation extracts rotational kinetic energy

↳ pulsar period increases over time

* Also hypothesizes that particle jets account for some/much pulsar slow-down (Graham Lyne & Graham Smith)

$$E = \frac{I\Omega^2}{2} = \frac{2\pi^2}{P^2}I, \quad I = \frac{2}{5}MR^2$$

$$\dot{E} = \frac{d}{dt}\left(\frac{I\Omega^2}{2}\right) = I\Omega\dot{\Omega}$$

α = unknown inclination angle between rotation & emission axis

$$\Omega = \frac{2\pi}{P} \rightarrow \dot{\Omega} = 2\pi\frac{\dot{P}}{P^2}$$

$$\dot{E} = \frac{4\pi^2 I\dot{P}}{P^3} \quad \text{(spin-down luminosity)}$$

$-\dot{E} \simeq P_{rad}$ to find a lower limit on mag. field strength $B > B\sin\alpha$ at surface.

$$\frac{-4\pi^2 I\dot{P}}{P^3} = \frac{2}{3c^3}(BR^3\sin\alpha)^2\left(\frac{4\pi^2}{P^2}\right)^2$$

$$B^2 = \frac{3c^3 I\dot{P}P}{2\cdot 4\pi^2 R^6\sin^2\alpha}$$

$$\left(\frac{B}{\text{Gauss}}\right) = 3.2\times10^{19}\left(\frac{P\dot{P}}{s}\right)^{1/2}$$

(characteristic mag. field of pulsar)

$P\dot{P}$ is constant as
$$P\dot{P} = \frac{8\pi^2 R^6 (B\sin\alpha)^2}{3c^3 I} \quad \text{(indep. of time)}$$

$\therefore P\dot{P} = P'\dot{P}'$

$P\,dP = P\dot{P}\,dt$

$$\int_{P_0}^{P} P \cdot dP = \int_0^{\tau} P\dot{P}\,dt = P\dot{P}\int_0^{\tau} dt$$

where P_0 is original period. τ - characteristic age.

$$\frac{1}{2}(P^2 - P_0^2) = P\dot{P}\tau$$

in the limit $P_0^2 \ll P^2$

$$\frac{1}{2}P^2 = P\dot{P}\tau$$

$$\therefore \tau = \frac{P}{2\dot{P}} \quad \text{(characteristic age of pulsar)}$$

☆ This is typically an over-estimate:

↳ newly formed pulsar with a small P_0 may be quite oblate.

↳ Initially slowed by emitting quadrupole gravitational radiation

Additional Information

- Pulse profiles are sh freq. dep. so larger bandwidths may cause problems when averaging over frequencies.